

Making sense of antenna VNA measurements with the aid of the Smith chart

Andy Summers, G4KNO

Vector Network Analysers past and present



HP8505A – First introduced in 1976. The first VNA I worked with during Industrial Training at Marconi in 1986. It had a wideband internal variable line-stretcher that allowed 'calibration' to the end of the measurement cable.

It cost about the same as a 3-bed semi-detached house in Chelmsford at the time!



HP8754A – First introduced in 1979 as a low cost alternative to the HP8505A. In my first proper job at Plessey Roke Manor (1988), this was the one you had to use if all the flashy newer ones were in use. It had no variable line stretcher, so was difficult to use.

Needless to say, for radio amateurs, owning either of these was a pipe dream!

Vector Network Analysers past and present



Fast-forward to today and there are several VNAs available to the radio amateur that won't require a re-mortgage.

They tend to be in the £300 - £500 range, but the NanoVNA retails for around £50! If anyone has experience of this device, I'd be interested to hear about it!



Some of the reasons for the availability of low cost VNAs are the availability of single-chip DDS sources, wide dynamic range integrated log detectors (e.g. AD8307), or low-cost SDRs.



If you're in the market for a VNA, try to get one that can tell you the sign of the reactance as well as its magnitude. My older MiniVNA can't, but it's not the end of the world as the results can be 'unwrapped' using something like [Zplots](#).

OK, so you've bought a VNA...

The context of this talk is about using a VNA to measure antennas. Presumably you've bought a VNA because you want to know more than just the VSWR? So what can a VNA tell us?

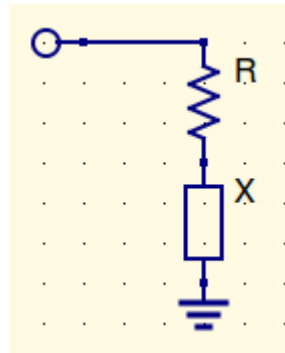
The whole point about using a VNA in antenna measurements is the ability to measure the resistance and reactance, and also know the sign of that reactance (i.e. whether it is inductive or capacitive). We do this at some plane of reference, say at the feedpoint.

Where your antenna is single element, single-band and full-size, it is likely not to be too far from 50R at resonance, a VSWR meter is probably all you need to tune it. However, when you need to do some 'matching', or want to know why things haven't quite worked out as expected, knowing the impedance is essential. Some instances where you may need to do this include the following:

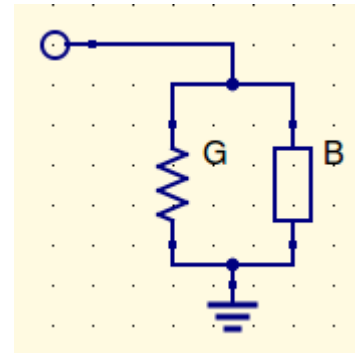
- Electrically short antennas
- The driven element of a parasitic array
- Multiband antennas

Nomenclature

We can describe a combination of resistance with capacitance or inductance in two different but equivalent ways:



≡



Series Impedance
 $Z = R + jX$ [ohms]
resistance reactance

Parallel Admittance
 $Y = G + jB$ [siemens | mho]
conductance susceptance

$$Y = \frac{1}{Z}$$
$$G = \frac{1}{R} \{X = 0\}$$

Sign change! →

$$B = -\frac{1}{X} \{R = 0\}$$
$$j = \sqrt{-1}$$

The 'j' operator represents a phase rotation of 90°, and so can mathematically describe how the current lags the applied voltage in an inductor, or leads it in a capacitor. A quantity that is not multiplied by j is said to be 'real', and where it is multiplied by j is said to be 'imaginary'. A 'complex number' has a real and imaginary part.

The importance of series-parallel equivalence

We can express the parallel network as resistance in parallel with reactance, so we can write:

$$Y = \frac{1}{Z} = \frac{1}{R_s + jX_s} \equiv G + jB$$
$$\frac{1}{R_s + jX_s} \equiv \frac{1}{R_p} - j\frac{1}{X_p}$$

We can do some complex algebra to figure out that:

$$R_p = \frac{R_s^2 + X_s^2}{R_s} \quad X_p = \frac{R_s^2 + X_s^2}{X_s}$$

This says that series resistance can be transformed to a larger value of parallel resistance dependent on the amount of series reactance. We could then add some parallel reactance of the opposite sign to cancel the net reactance.

This is the basis of all impedance matching, which always involves series followed by parallel reactances, or vice versa.

To design matching networks mathematically requires knowledge of complex number algebra and even then is time-consuming and prone to error.

This is where the Smith Chart comes in.. 😊

The Impedance Smith Chart Transformation

The Smith Chart is derived from the mathematical transformation that gives the normalised reflection coefficient, Γ , which is what your VNA actually measures

$$\Gamma = \frac{Z - Z_0}{Z + Z_0}$$

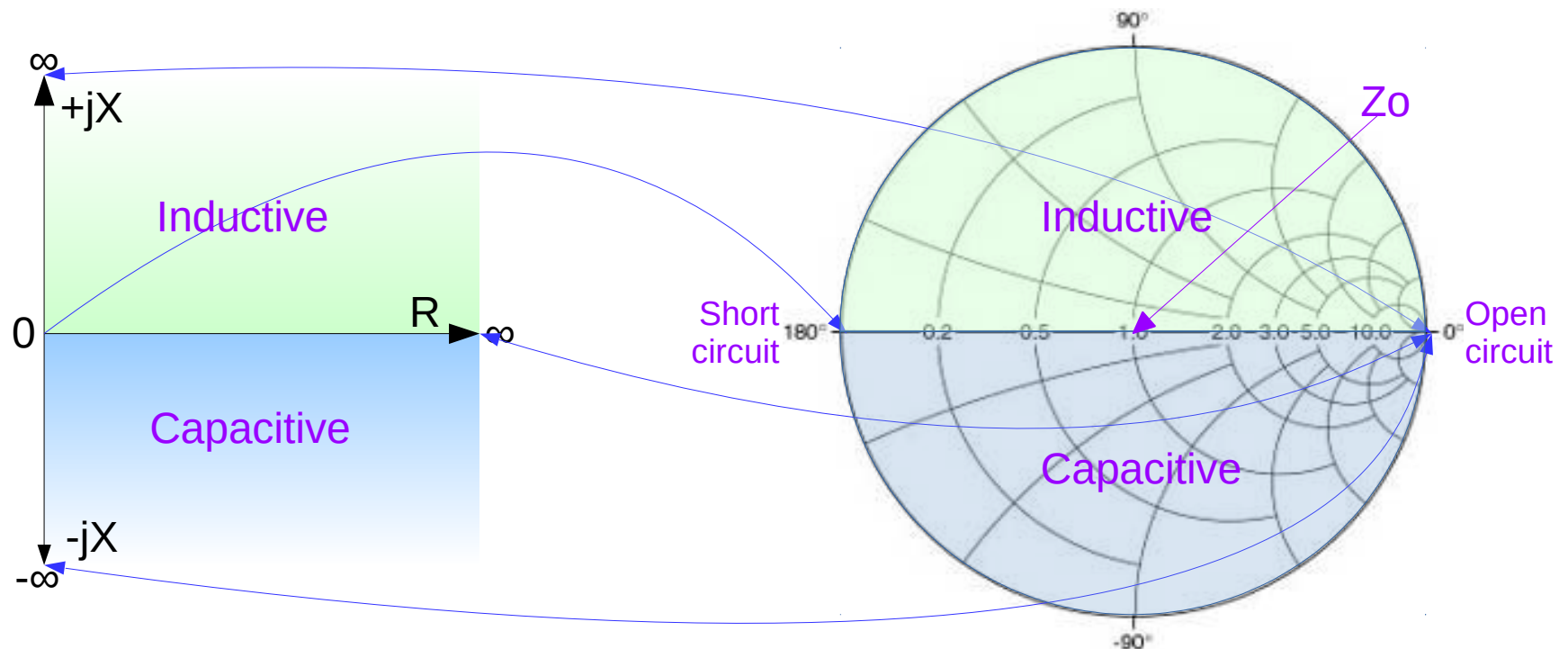
Γ is complex if Z is complex

The magnitude of Γ ranges over $0 \sim 1$ for all Z

The angle of Γ ranges over $-180^\circ \sim +180^\circ$ for all Z

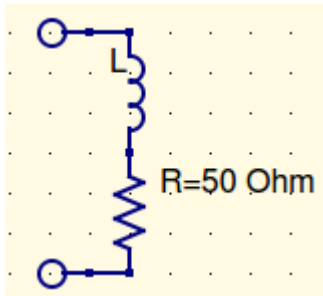
VSWR is related to Γ : $VSWR = \frac{1 + |\Gamma|}{1 - |\Gamma|}$

Notice that VSWR is scalar quantity ranging from $1 \sim \infty$



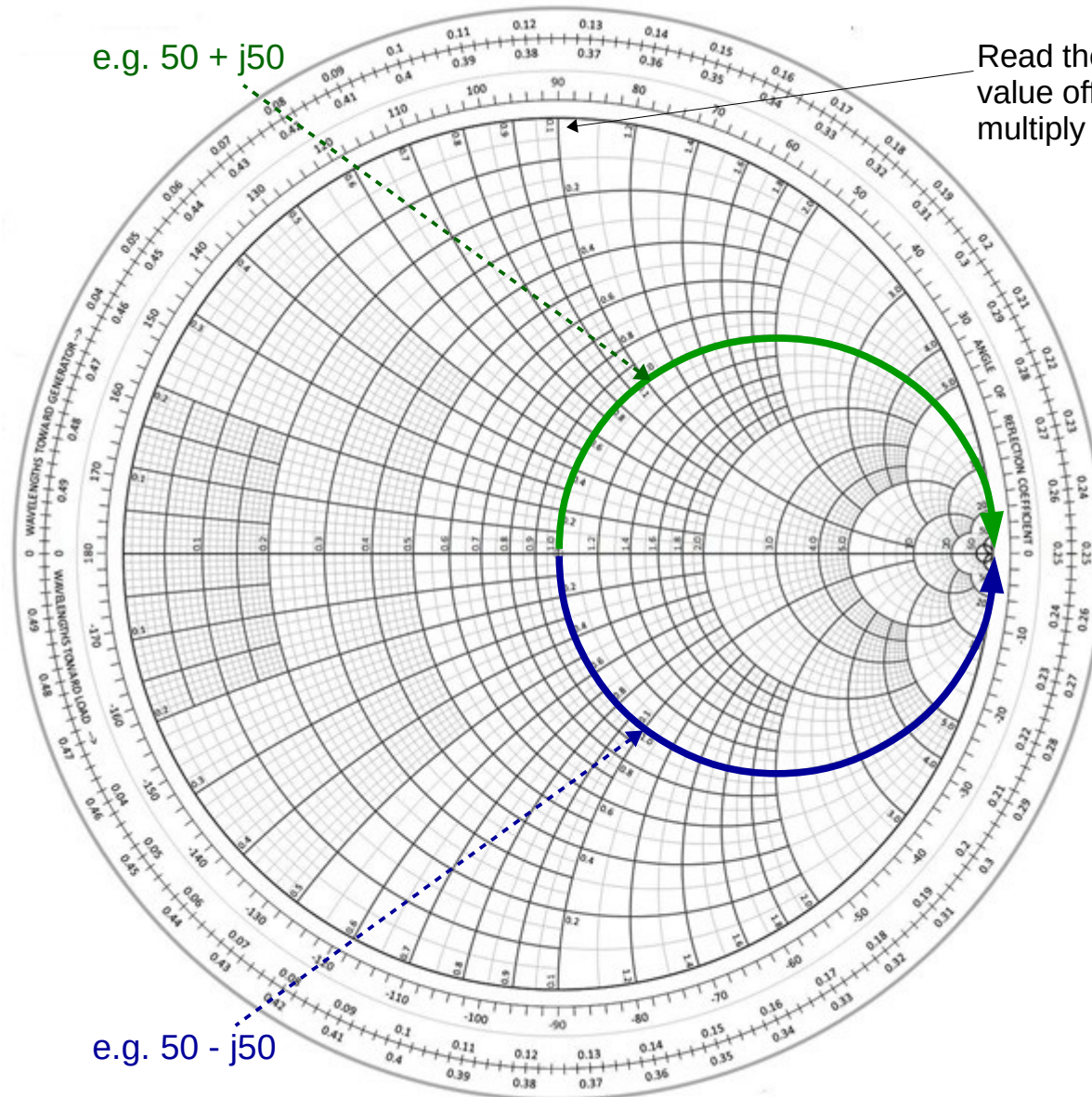
Series R,L,C

For constant R, a series R,L,C circuit moves on the lines of constant resistance:



e.g. $50 + j50$

Read the normalised reactance value off the outer ring, then multiply by Z_0 .

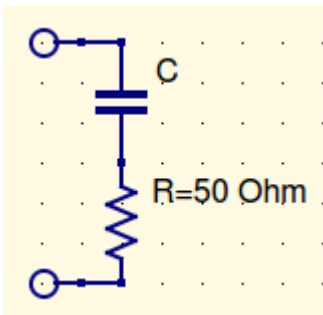


$$X_L = +j2\pi fL$$

Increasing inductive reactance ($L \uparrow, f \uparrow$)

Increasing capacitive reactance ($C \downarrow, f \downarrow$)

$$X_C = -j \frac{1}{2\pi fC}$$



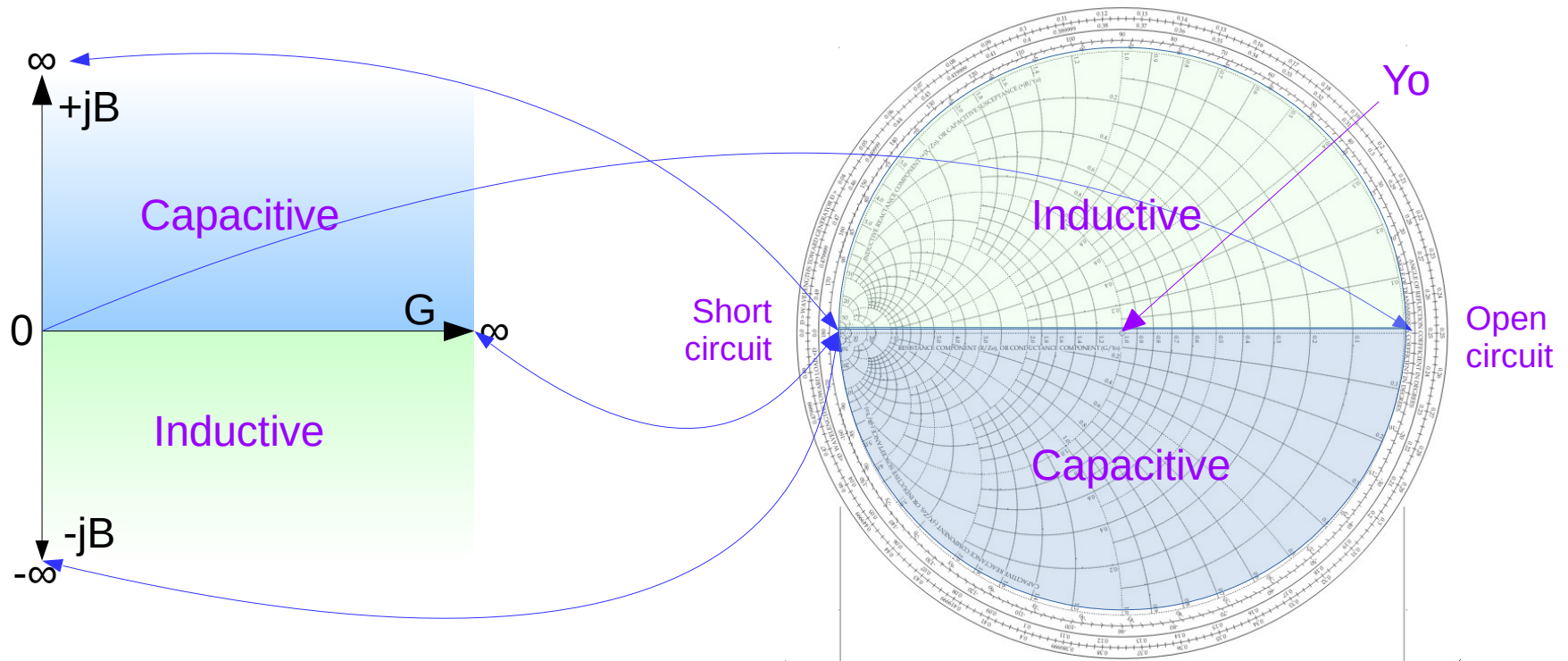
e.g. $50 - j50$

The Admittance Smith Chart

We can do the same transformation for admittance (i.e. parallel circuits):

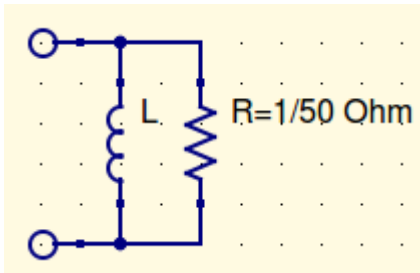
$$\Gamma = \frac{Z - Z_0}{Z + Z_0} = \frac{\frac{1}{Y} - \frac{1}{Y_0}}{\frac{1}{Y} + \frac{1}{Y_0}} = \frac{1 - \frac{Y}{Y_0}}{1 + \frac{Y}{Y_0}}$$

The main thing to recognise is that the result is the impedance Smith Chart flipped horizontally (or rotated). Despite the sign change of the imaginary part, open, short, Z_0 (Y_0), which half is inductive and which is capacitive have all remained in the same positions.

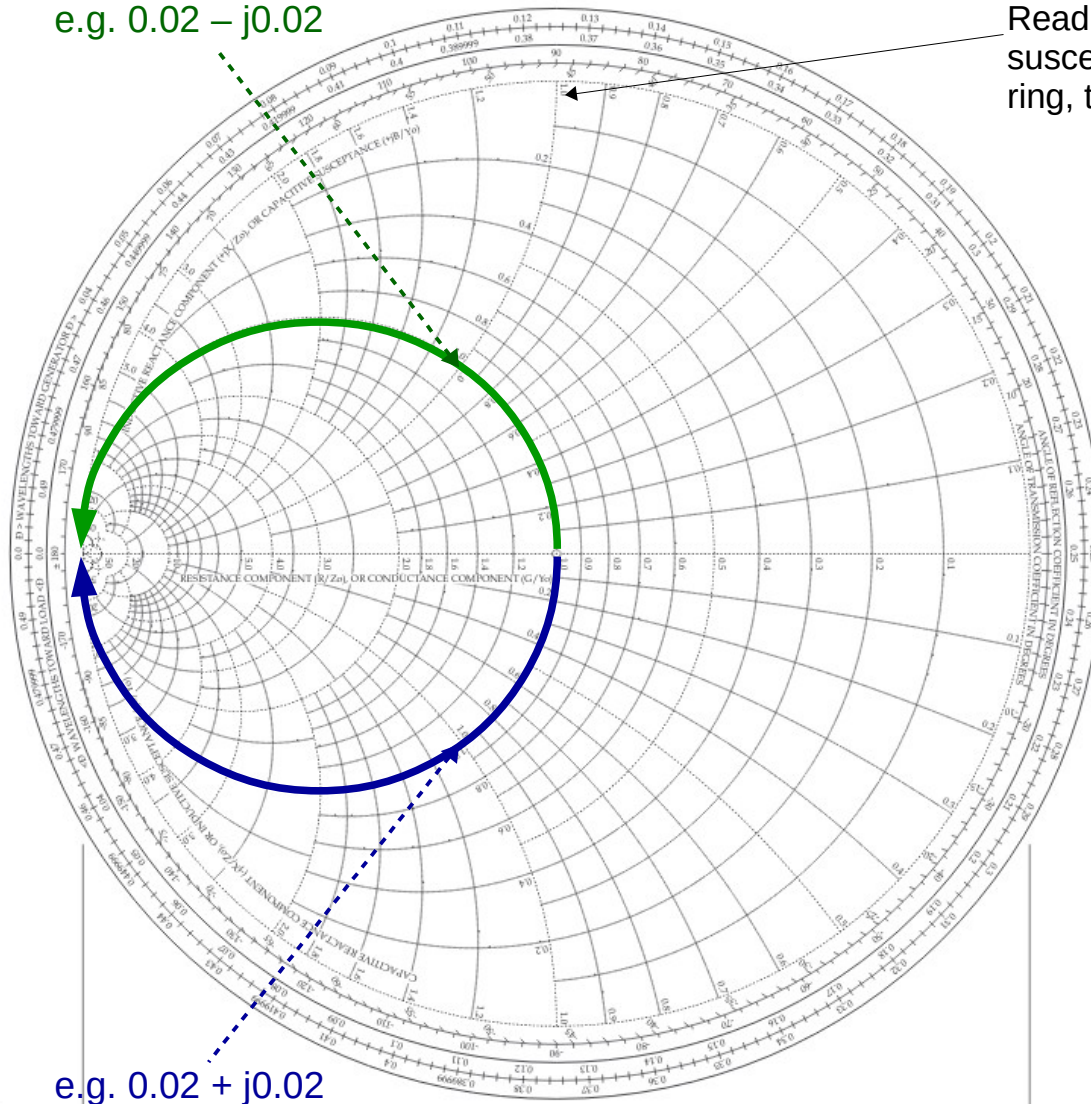


Parallel G,L,C

For constant G, a parallel G,L,C circuit moves on the lines of constant conductance:



e.g. $0.02 - j0.02$



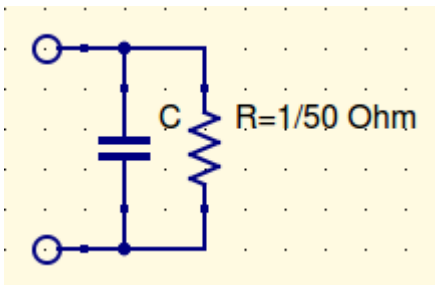
Read the normalised susceptance value off the outer ring, then multiply by $Y_0 (=1/Z_0)$.

$$B_L = -j \frac{1}{2\pi f L}$$

Increasing inductive
susceptance
($L \downarrow, f \downarrow$)

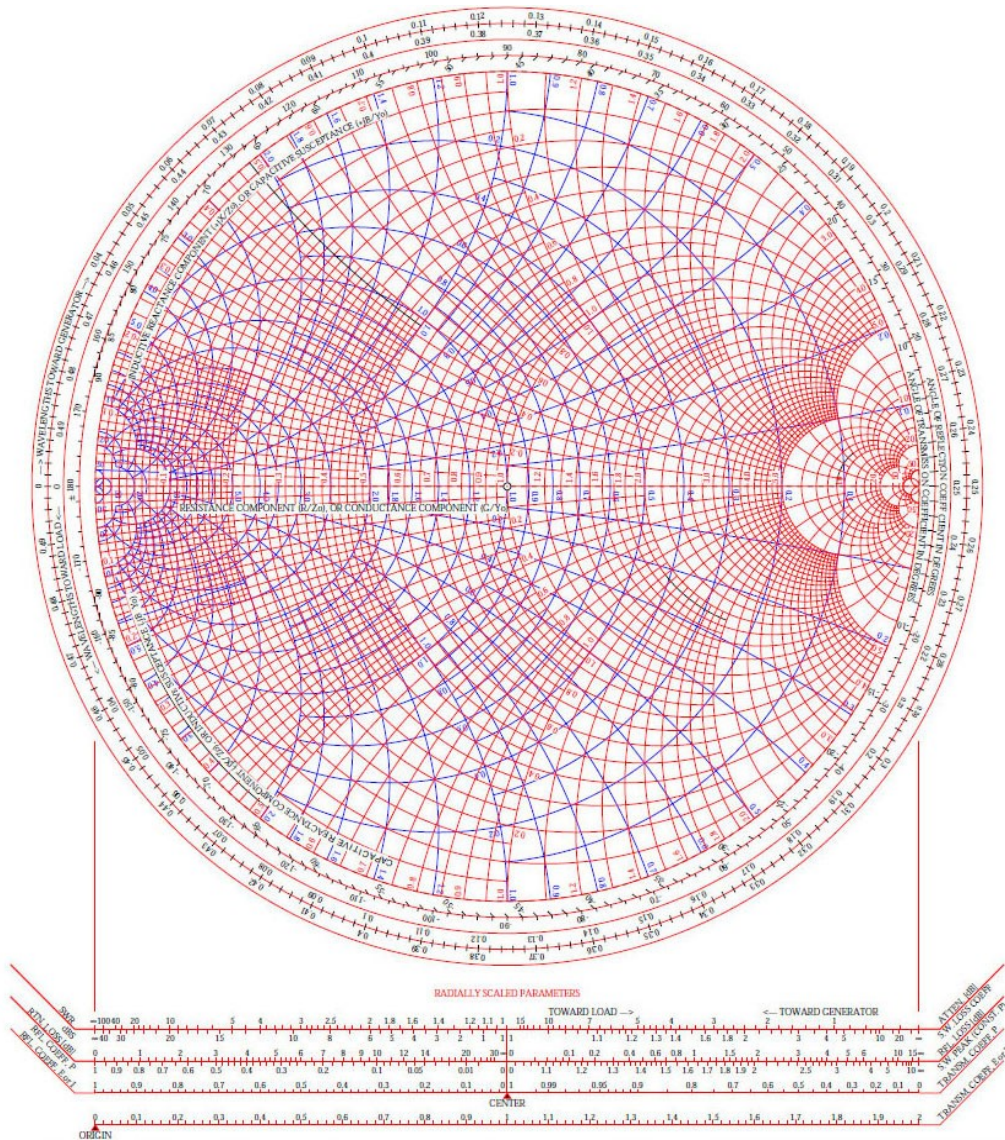
Increasing capacitive
susceptance
($C \uparrow, f \uparrow$)

$$B_C = j2\pi f C$$



e.g. $0.02 + j0.02$

The Immittance Smith Chart



An immittance Smith Chart allows us to plot impedance and admittance on a single chart.

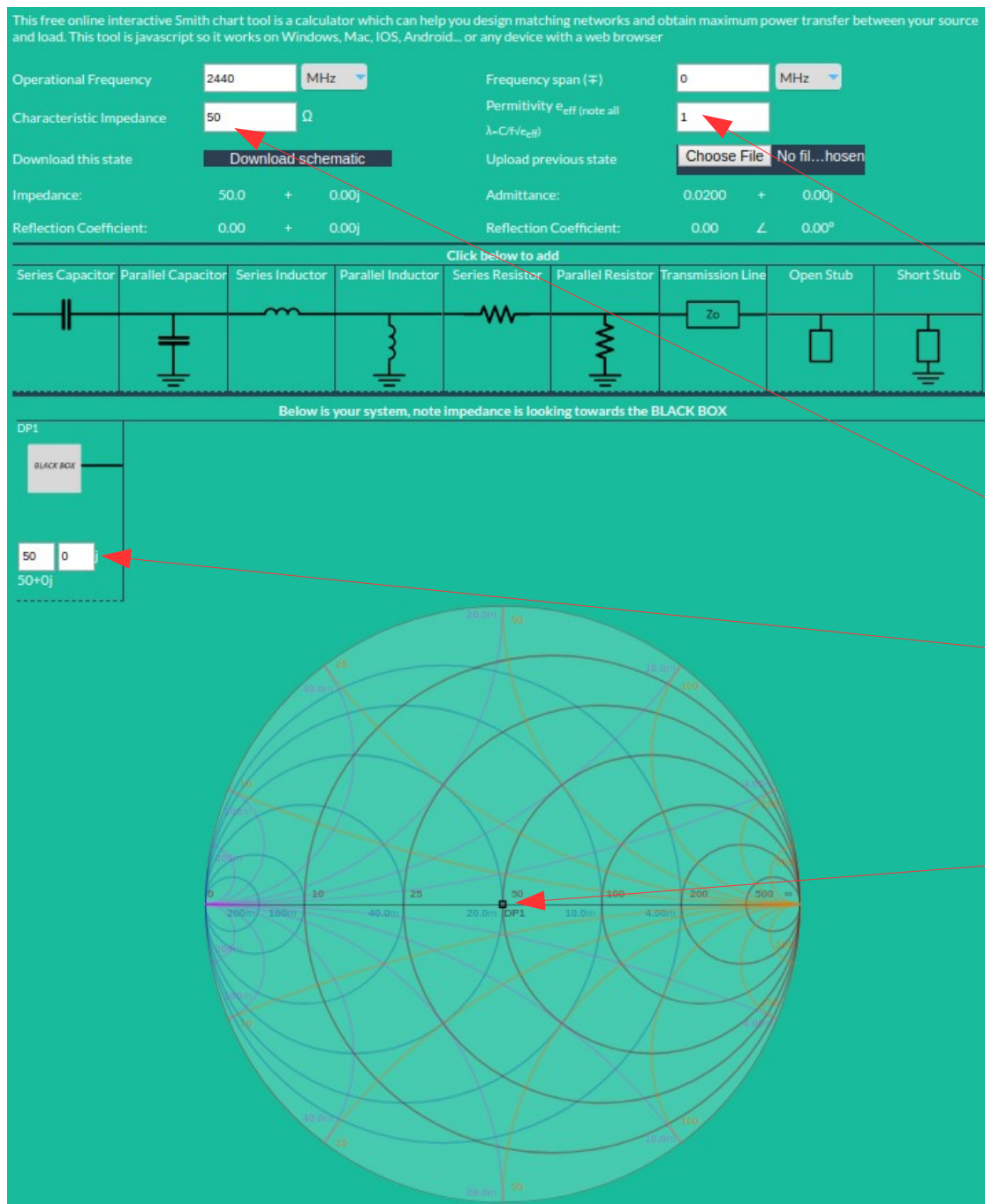
Conversion between impedance and admittance is instant (just read off on the appropriate grid).

It allows us to follow the contours traversed in series-parallel matching networks with ease.

'Old school', we would design matching networks graphically, then read off the lengths of the arcs as reactances or susceptances, de-normalise and calculate the capacitance and inductance values.

But from here on, I recommend you use a Smith Chart program to make things even easier...

An On-Line Smith Chart



There are numerous Smith Chart apps, but my new favourite is on-line at:
https://www.will-kelsey.com/smith_chart/

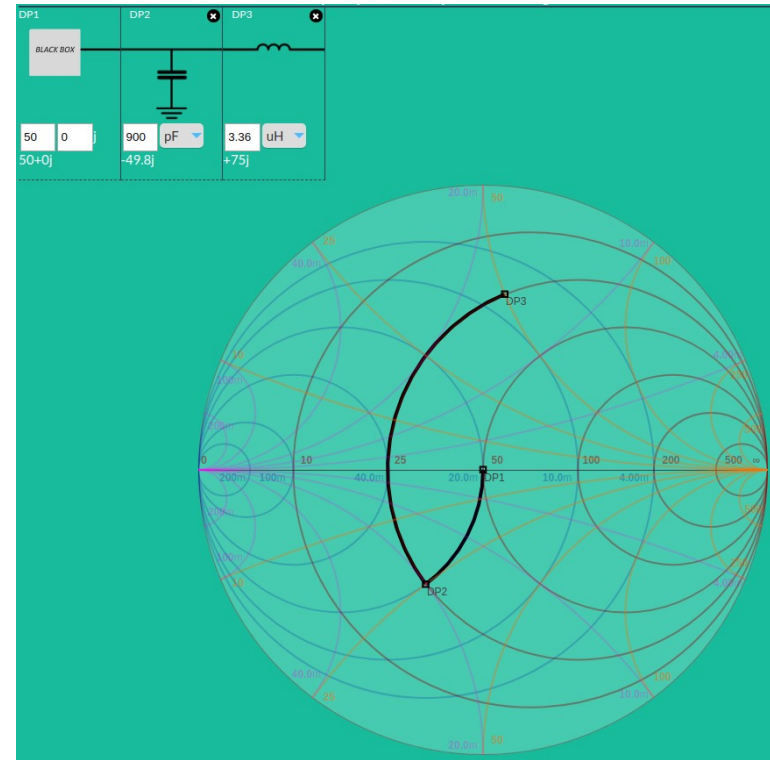
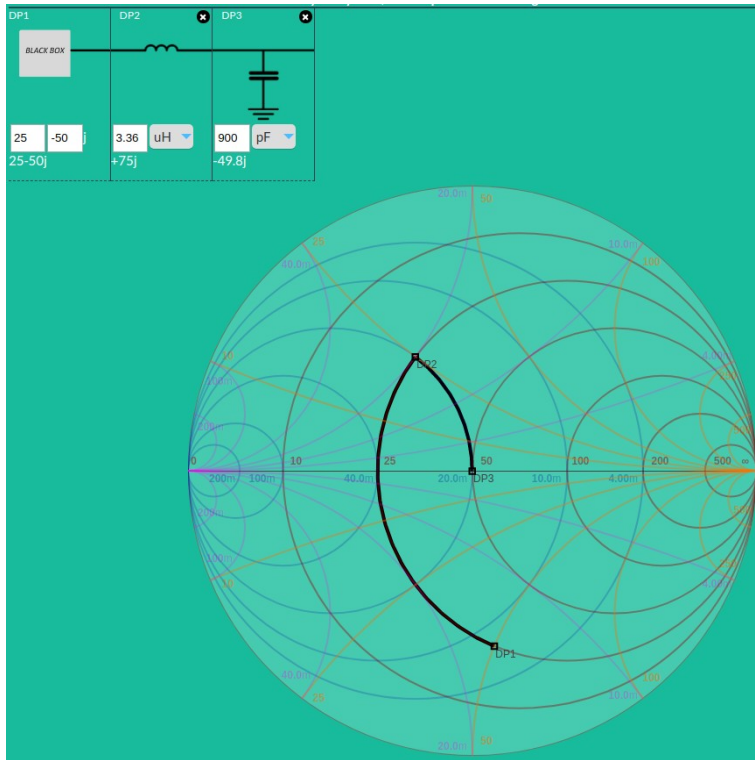
To plot transmission lines change the value of ϵ_{eff} to $1/(\text{velocity factor})^2$, e.g. $1/0.7^2 \approx 2$.

The Smith Chart displayed has already done the de-normalisation, so the centre is 50R rather than 1.0. You can change the normalised impedance if you want.

Enter a start impedance here, e.g. your antenna impedance. Here it is defaulted to 50R, so you just get a single point in the middle.

Add components as necessary to design your matching network.

A word about conjugate matching

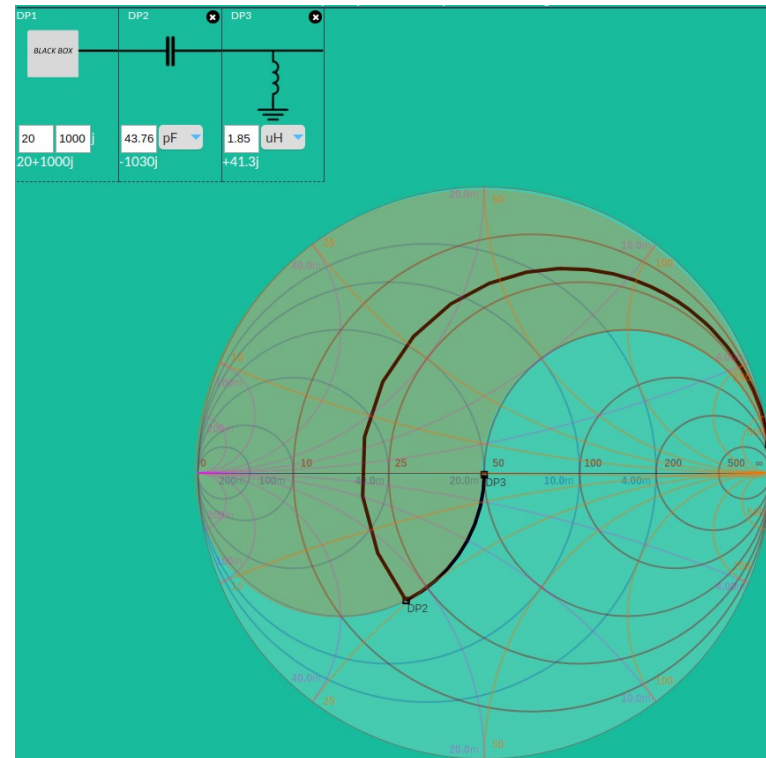
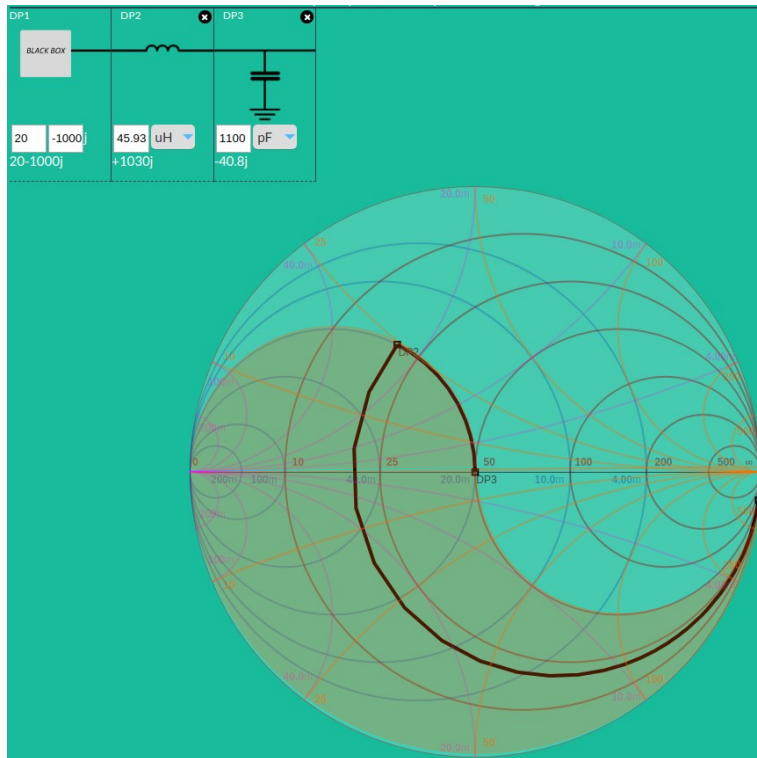


- Start with the impedance to be matched ($25 - j50$ in this example @3.55MHz).
- Design the matching network to get to $50R$.

- Start at $50R$.
- Design the matching network to get to the complex conjugate of the impedance to be matched (just change the sign of the reactance).

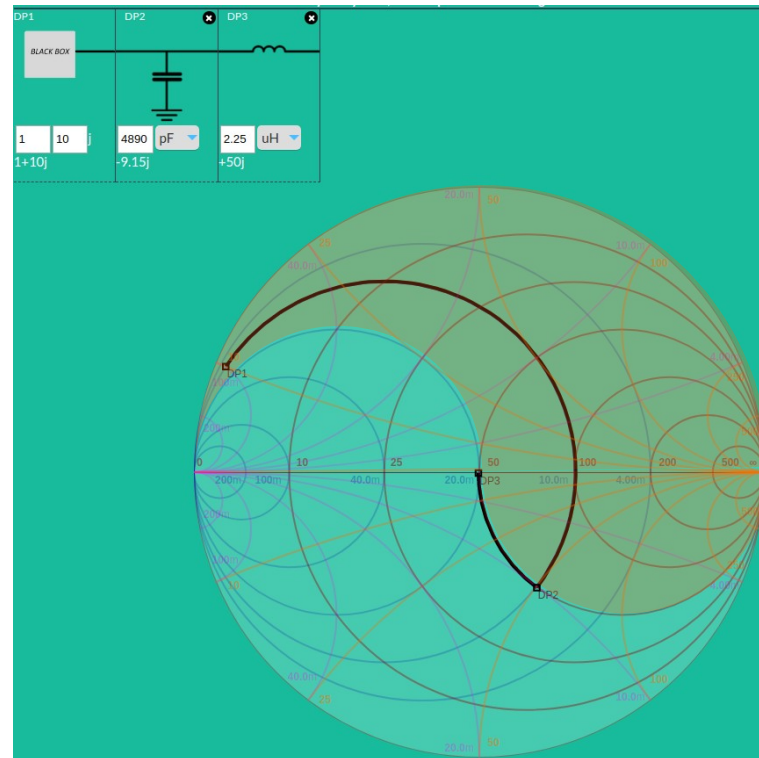
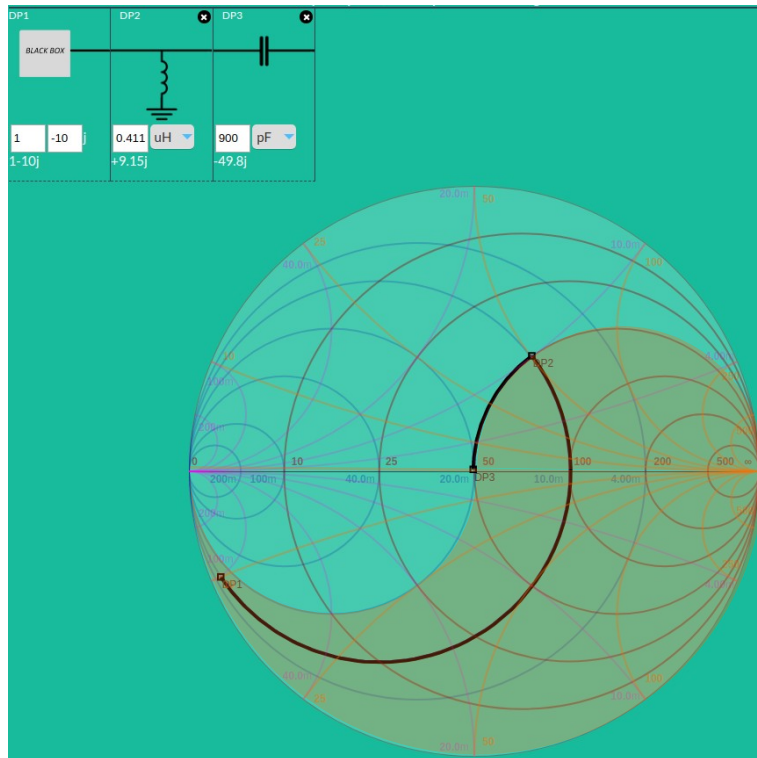
The method on the left is more intuitive, but sometimes the method on the right is useful.

The four minimum-Q matching networks (1)



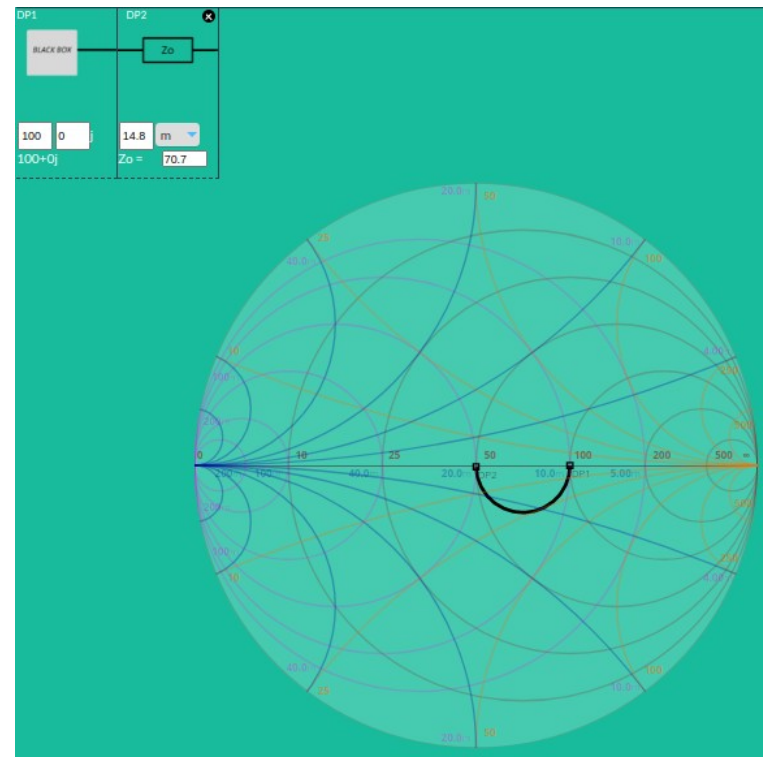
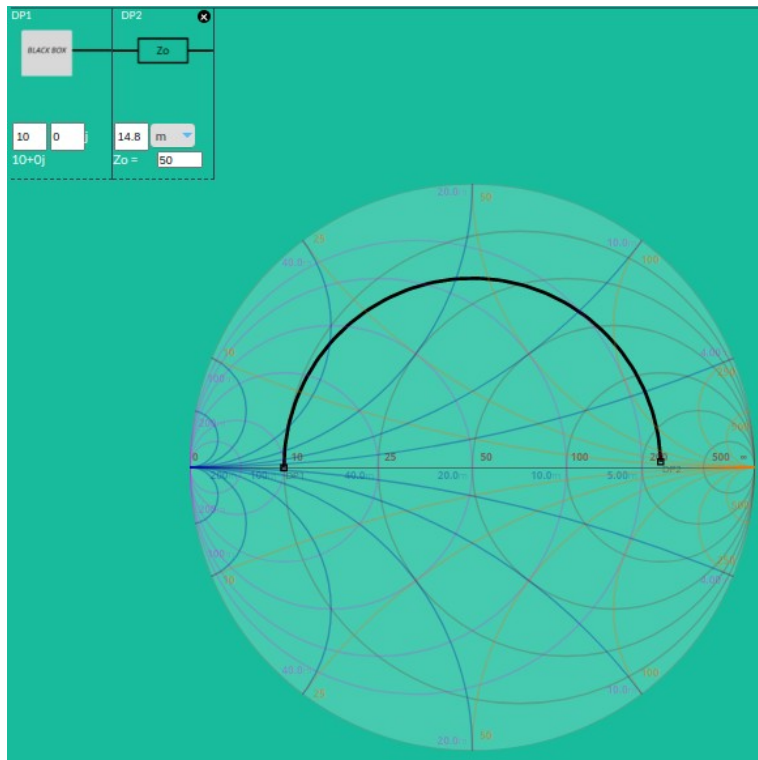
These two matching networks can't match inside the unit circle of impedance ($>50R$).

The four minimum-Q matching networks (2)



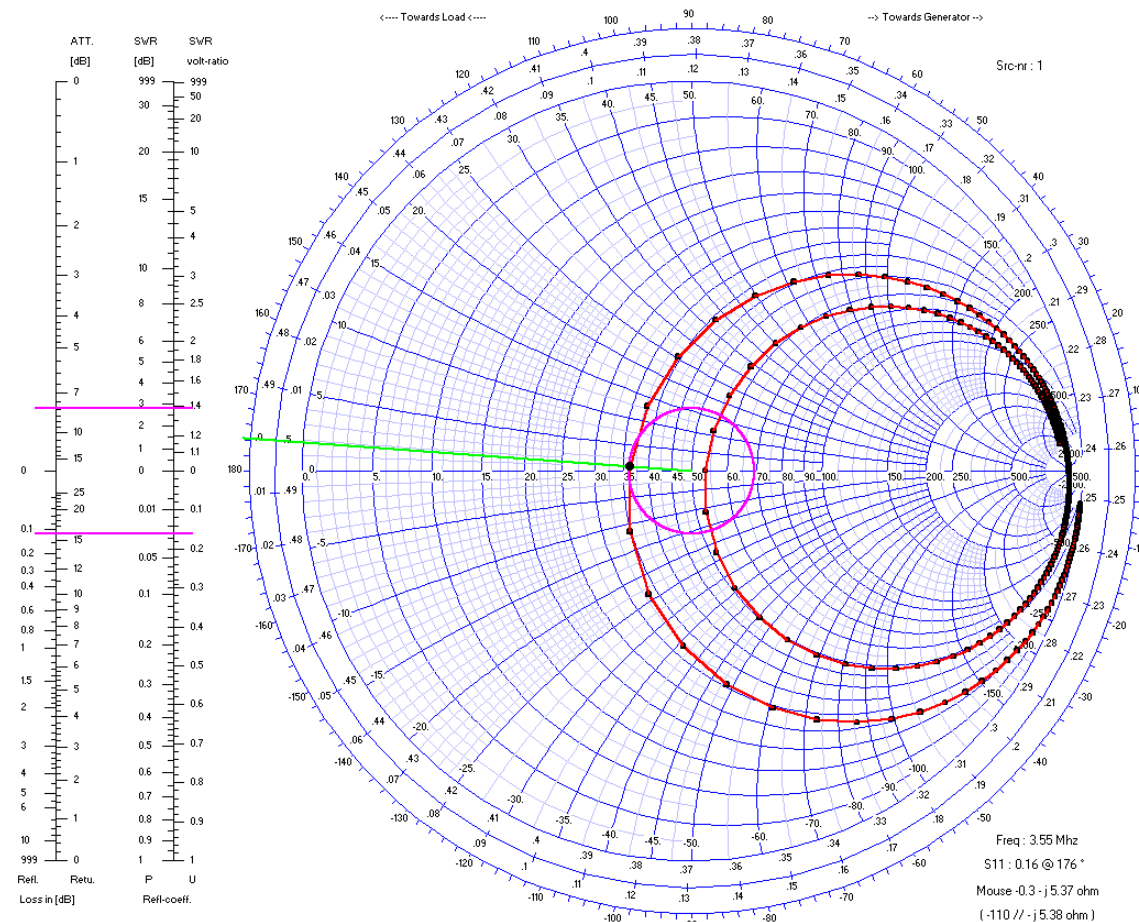
These two matching networks can't match inside the unit circle of admittance ($<50R$).

Transmission Lines



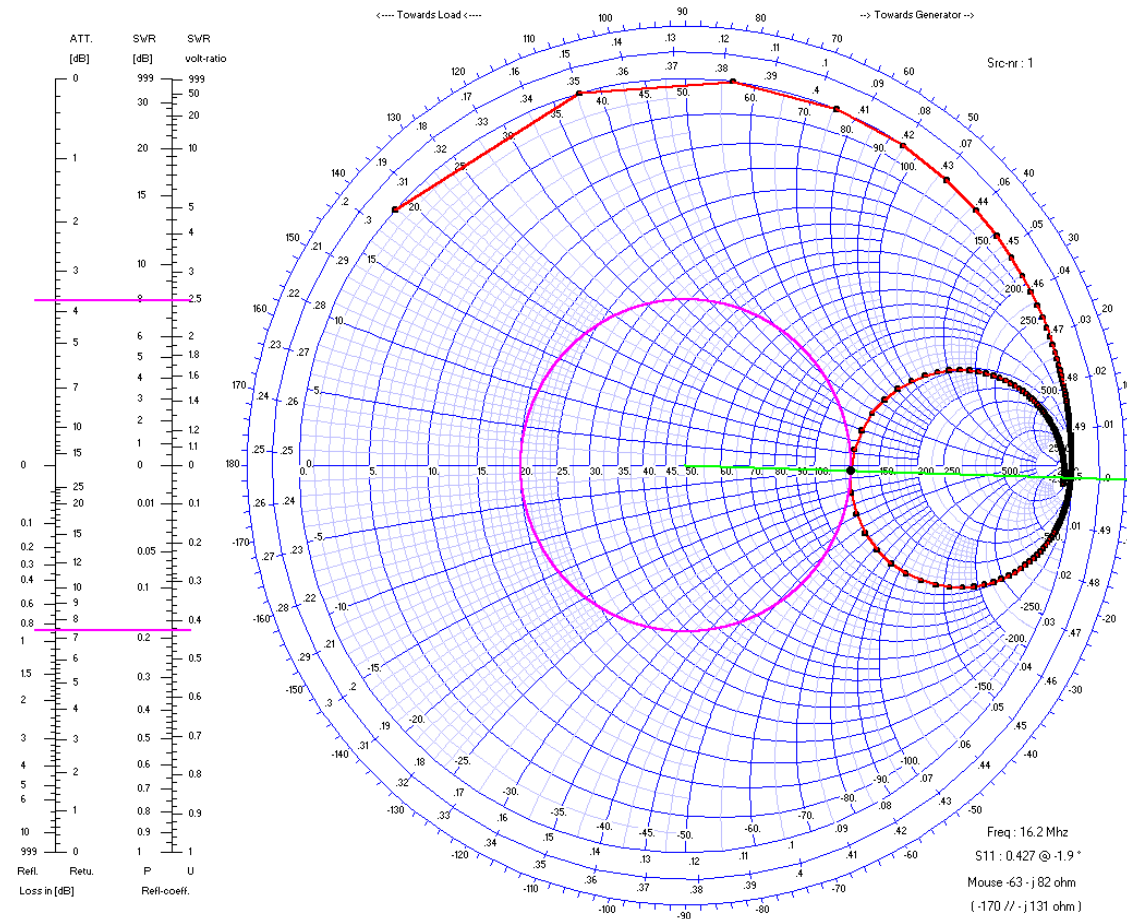
- A $\frac{1}{4}$ -wavelength of transmission line (14.8m @3.55MHz with $V_f=0.7$) rotates an impedance *clockwise* around the Z_0 of the transmission line.
- Note that 90° electrical length is 180° on the Smith Chart.
- You might recognise the right-hand example as a 2:1 $\frac{1}{4}$ -wave transmission line transformer.

Monopole impedance vs. frequency



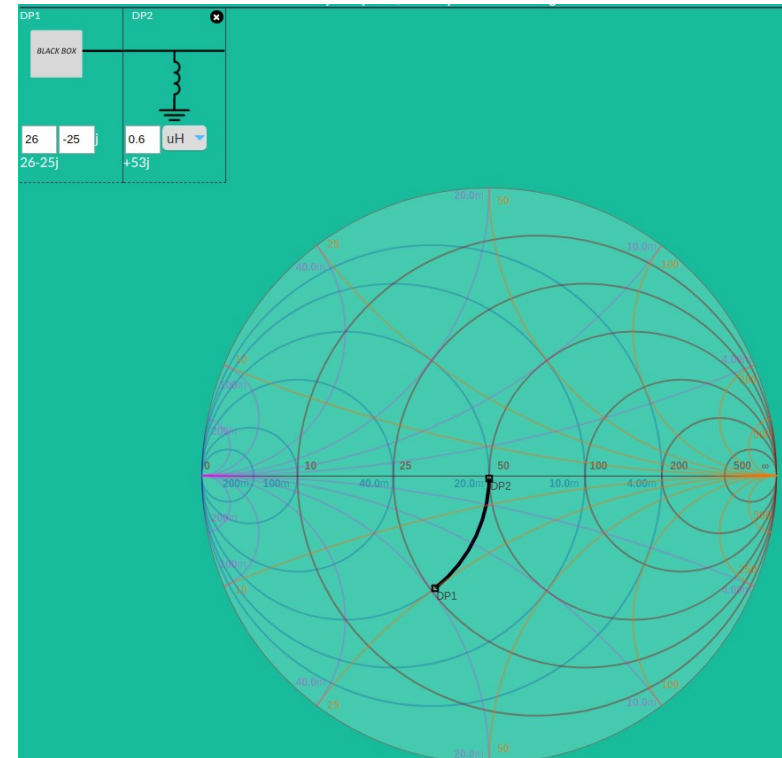
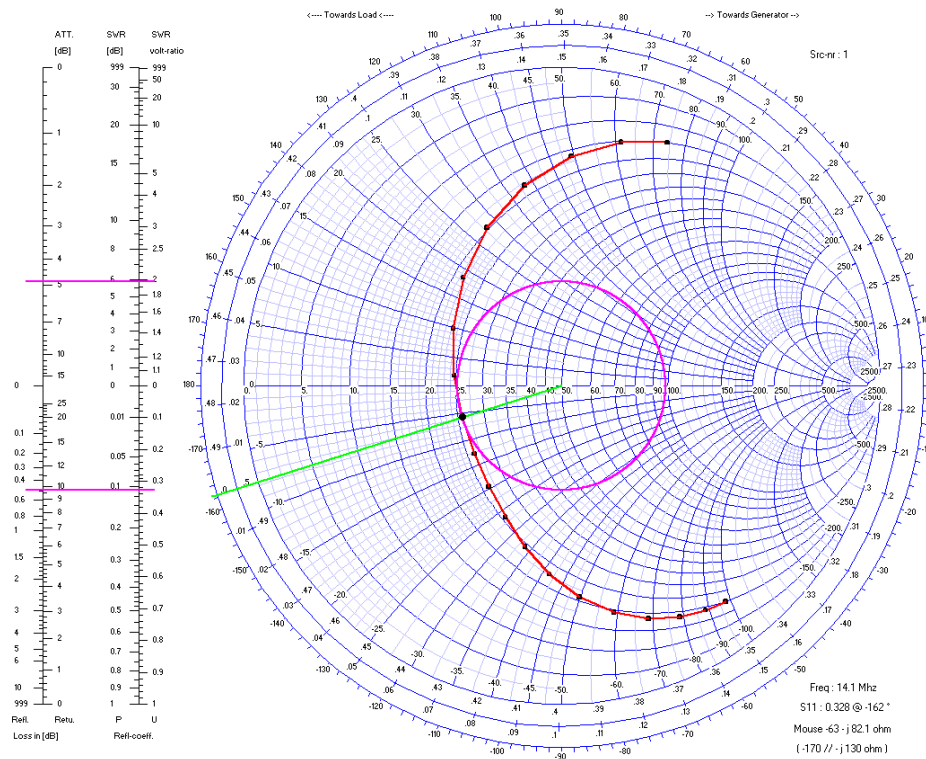
- Here I'm using 4NEC2 www.qsl.net/4nec2/ to model a 20.6m monopole over ideal ground.
- 4NEC2 can display impedance results over a sweep of frequencies (1-13.7MHz here) on a Smith Chart!
- Notice that at low frequencies Z tends to an open-circuit, 'resonates' close to the expected 36.5R, then goes on to become almost an open circuit when $\frac{1}{2}$ -wave long.
- Also notice how the resistive part is increasing, which will help real-world efficiency.
- A good match to 50R is obtained at a length of around $\frac{3}{4}$ -wave.
- If we kept going up in frequency, the circles would get smaller and tend to a resistance ($\sim 600R$).

Loop impedance vs. frequency



- This is a quad loop antenna in free-space, with 2.44m sides, swept over 0.1-25.5MHz.
- At low frequencies, Z tends to a short-circuit.
- We get high-Z when the total wire length is $\frac{1}{2}$ -wave.
- At 1-wave we get a resistive part close to the expected value of 120R.
- At higher frequencies we also get ever-decreasing circles tending to a similar resistance.

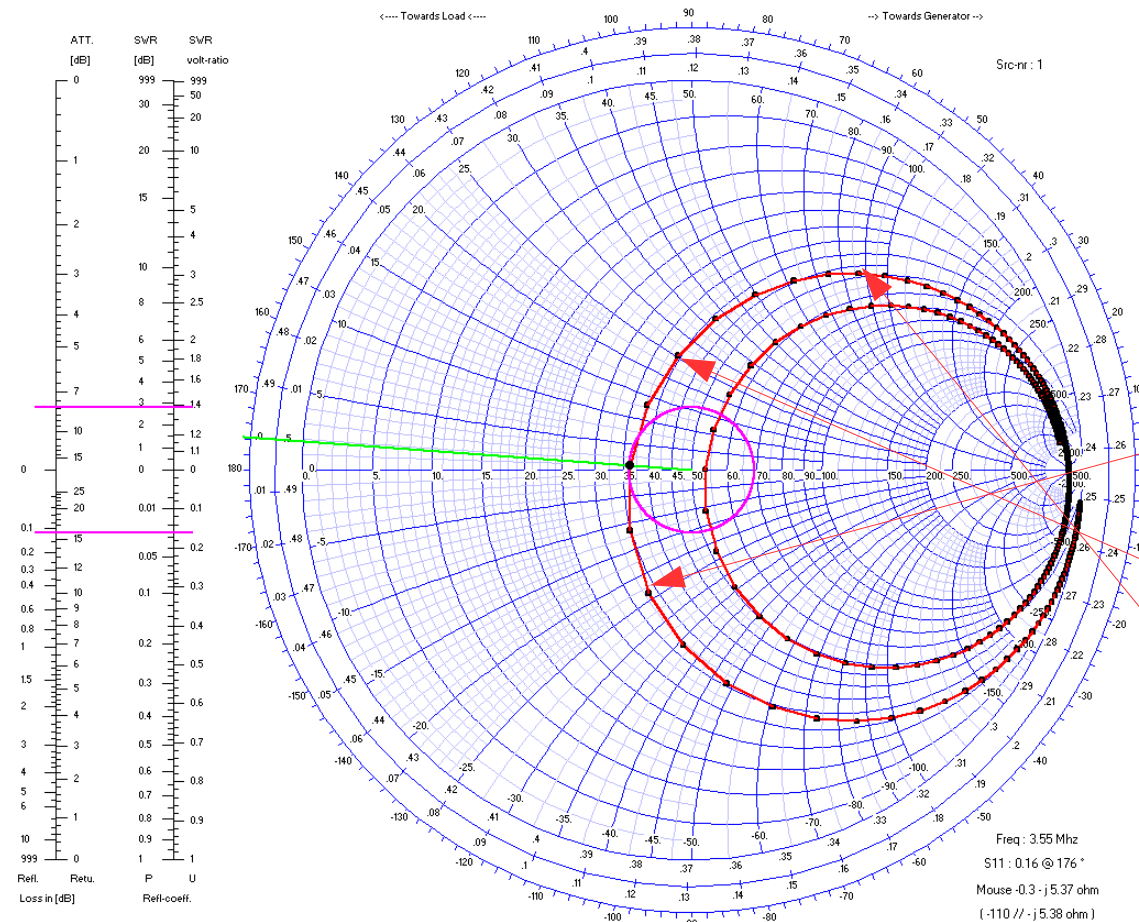
A Hairpin match on a 3-ele beam



- A 3-ele 14.1MHz Yagi has a feedpoint impedance of $\sim 25R$, but you can see now how if we make the driven element slightly shorter we can get on the 0.2S constant conductance circle and use just shunt inductance to get to 50R.
- This is commonly done with a U-shaped piece of aluminium.



Matching options on a monopole



- From the previous Smith Chart plot of an 80m monopole, we have three single element matching options:
 - Make the antenna slightly shorter than resonance and add shunt inductance (a.k.a. a hairpin match); about 3.2uH.
 - Make the antenna slightly longer than resonance and add shunt capacitance; about 460pF.
 - Make the antenna even longer and add series capacitance; about 450pF.

- The shunt-C option should provide the highest bandwidth because it has the lowest ratio of X/R (=Q).

A Case Study (1)

6-4 THE W6SAI HF ANTENNA HANDBOOK

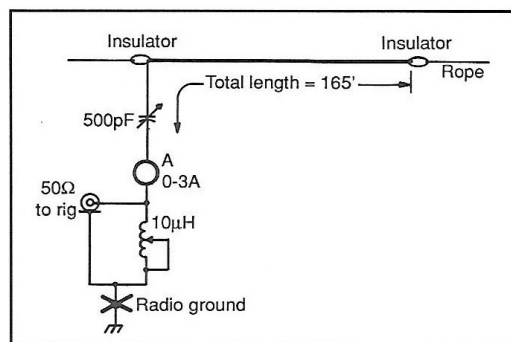
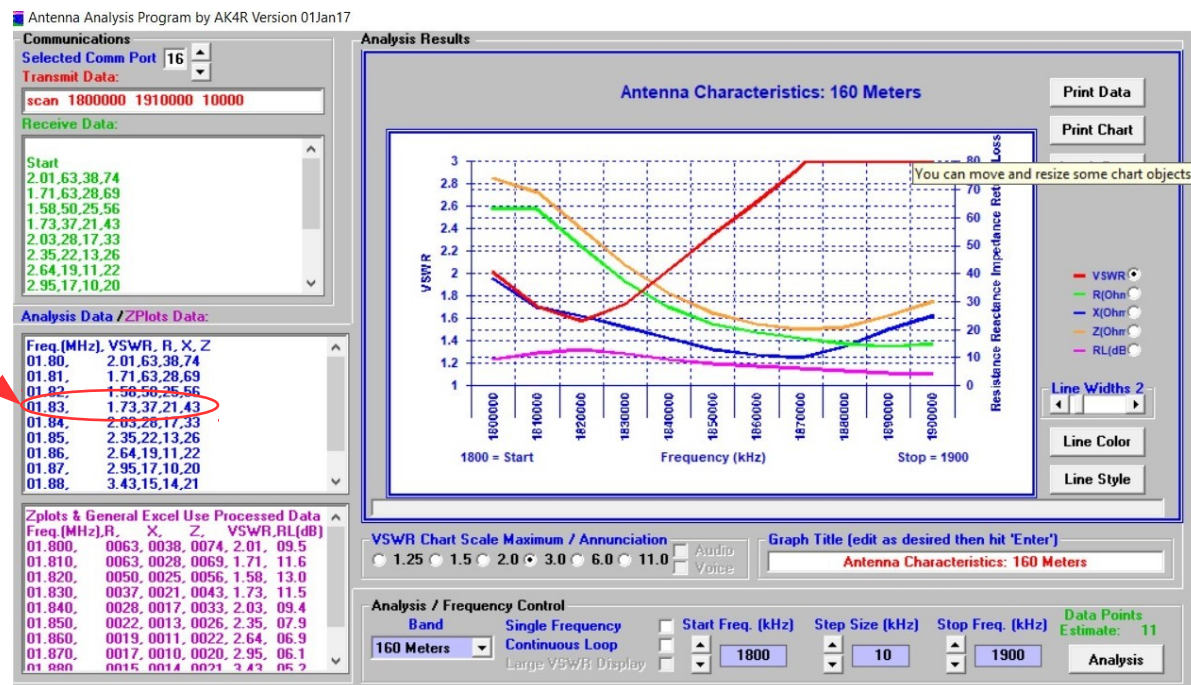


Figure 6-5. The series-tuned Marconi for 160 meters depends on a good radio ground to function properly. Don't let the power distribution ground get mixed up with the radio ground.

$37 \pm j21$

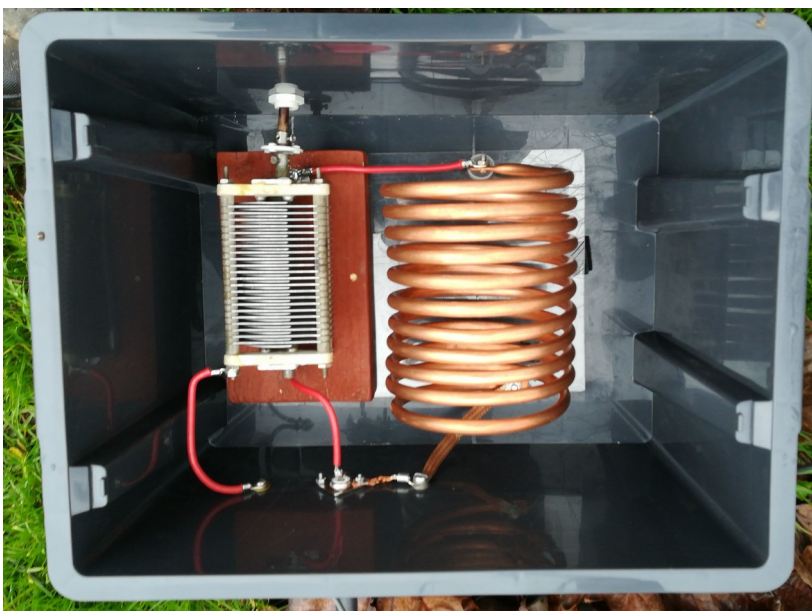


From Chris, G3SJJ:

"The total length is 50m of which 17.5 is vertical. L is approx 7 uH and C is around 250pF.

Using the Sark Mini60 antenna analyser I get the attached results. That is at the end of about 50m of RG214 which goes into the shed at the base of my main Tennamast.

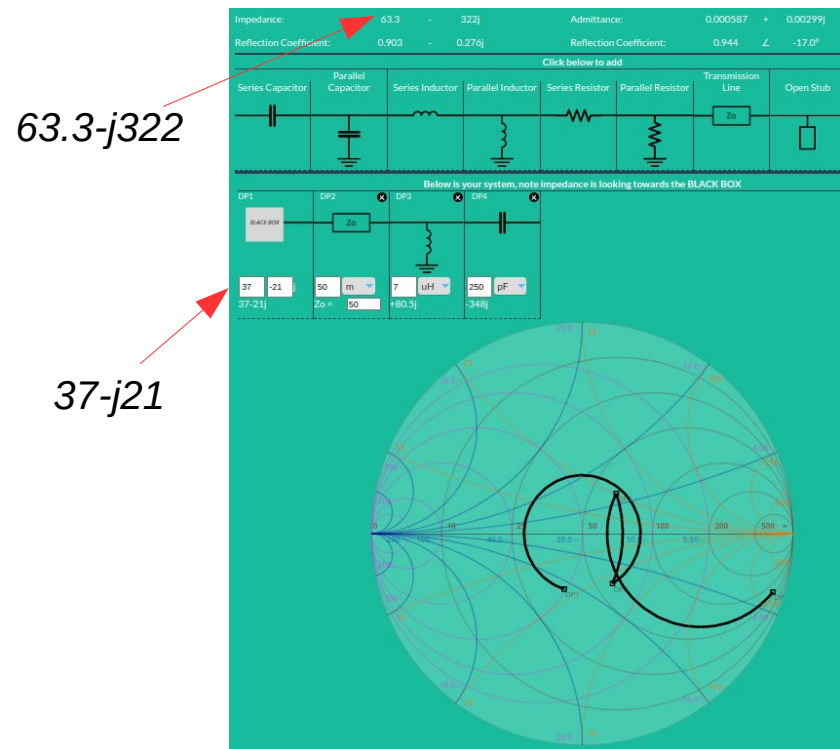
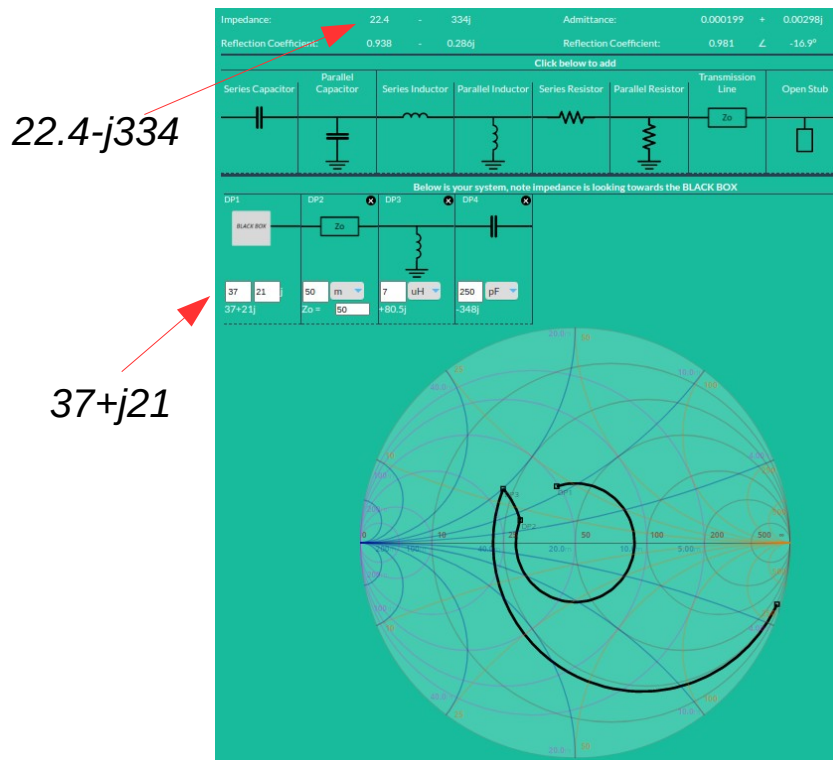
I wonder if reducing the number of turns by 1 or 2 then re-dipping the C would get the swr lower and give me a bit more bandwidth?"



A Case Study (2)

Ideally, we want to know the *feedpoint* impedance of Chris's inverted-L, so we can figure out whether reducing L can help get a better match. But actually we have all the information to be able to do that.

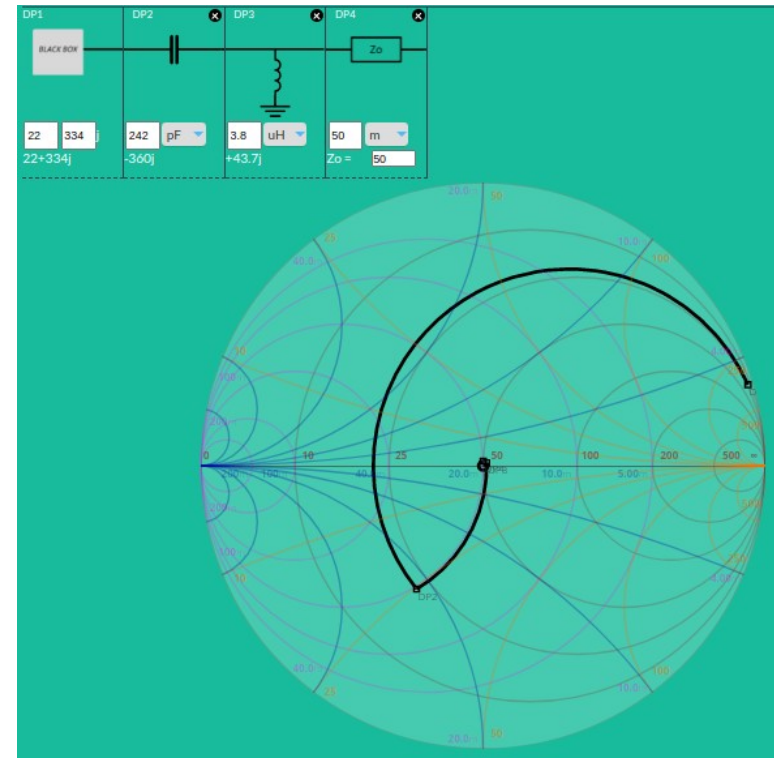
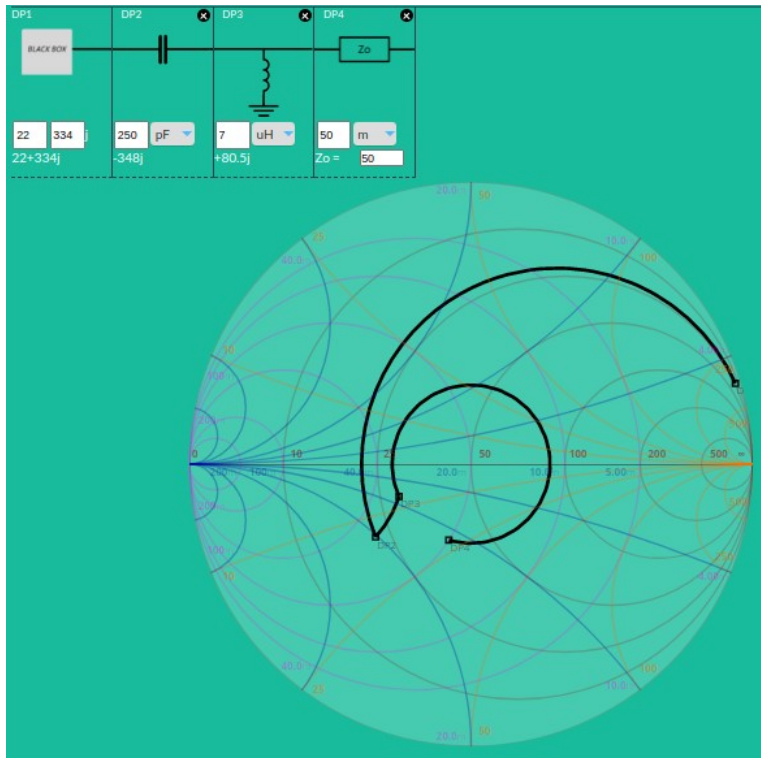
We start with what Chris measures at the shack end, and enter the transmission line, shunt-L and series C, which will give us the conjugate of the antenna impedance. We need to do this for either possibility of reactance sign.



Because it's an inverted-L, we know that the resistive part should be $\ll 36R$, so the most plausible result is the one on the left. And because this is the conjugate of the antenna feedpoint impedance, said impedance is $22+j334$ (i.e. inductive – as expected). Chris also confirmed this is not far from what MMANA_GAL predicts at $21+j223$, although the reactance is a bit out.

A Case Study (3)

Now we can start with the known antenna impedance and put the matching components the right way round (left hand plot). Varying C doesn't get you much closer to 50Ω – as Chris discovered.



On the right hand plot, C needs to be reduced to 242 pF and L needs to be reduced to 3.8 uH to achieve a match at 1.83 MHz . Since inductance scales as N^2 (a bit less for an air-cored inductor), Chris needs to take a couple of turns off his inductor.

As discussed earlier, better bandwidth might be obtained by reducing the length.

Questions?